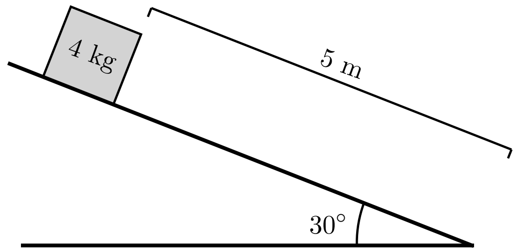
Honors Physics | Honors Physics Final Exam | Name: Answer Key
Spring, 2018 | | Mr. Leonard

Instructions: **The Exam is divided into two parts: a 35-question multiple-choice section and a four-question free-response section; the two sections are weighed equally.**

Use the blank spaces provided on the exam to fully answer the following questions. SHOW ALL OF YOUR WORK.

Note: You may continue to use $g = 10\,\mathrm{m/s^2}$ and $k = 9(10^9)\,\mathrm{N\cdot m^2/C^2}$ if you prefer.

1. A 4 kg block slides down a 10 m long ramp inclined to an angle to 30°.



(a) How much energy is initially stored in the system?

Solution: let d be the length of the ramp. From trigonometry, we see that the initial vertical height h of the box is $h = d\sin(30^\circ)$. Plugging this into the formula for gravitational potential energy gives:

$$\begin{aligned} PE_{\mathrm{g}} &= m\,g\,h \\ &= m\,g\,d\,\sin(30^\circ) \\ &= (4\text{ kg})\left(10\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(5\text{ m})\frac{1}{2} \\ &= 100\text{ J} \end{aligned}$$

(b) Assuming a frictionless surface, how fast would the box be moving when it reaches the bottom of the ramp?

Solution: If the ramp is frictionless we can use conservation of energy.

$$\begin{aligned} E_0 &= E_{\mathrm{f}} \\ E_0 &= \frac{1}{2}m\,v^2 \\ v &= \sqrt{\frac{2\,E_0}{m}} \\ &= \sqrt{\frac{2(100\text{ J})}{4\text{ kg}}} \\ &= 7.1\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

(c) Due to friction, the box reaches the bottom of the ramp with a speed of only 5 $\frac{\mathrm{m}}{\mathrm{s}}$. How much work was done by friction?

Solution: Because friction is a non-conservative force, we can find the work done by friction, by calculating the change in mechanical energy.

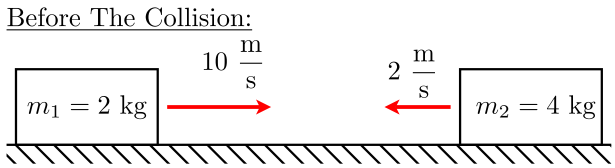
$$\begin{aligned}
W_\mathrm{f} &= E_\mathrm{f} - E_0 \\
&= \frac{1}{2}\, m\, v^2 - E_0 \\
&= \frac{1}{2}(4\ \mathrm{kg})\left(5\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 - 100\ \mathrm{J} \\
&= 50\ \mathrm{J} - 100\ \mathrm{J} \\
&= -50\ \mathrm{J}
\end{aligned}$$

(d) What was the average frictional force exerted on the box as it slid down the ramp?

Solution: Using $W = F\Delta x \cos\theta$, we can solve for F.

$$\begin{aligned}
W_\mathrm{f} &= F\,\Delta x \cos\theta \\
W_\mathrm{f} &= F\,\Delta x \xcancel{\cos(180^\circ)}^{-1} \\
F &= \frac{-W_\mathrm{f}}{\Delta x} \\
&= \frac{50\ \mathrm{N}}{5\ \mathrm{m}} \\
&= 10\ \mathrm{N}
\end{aligned}$$

2. A 2 kg block slides to the right across a frictionless surface with an initial speed of 10 $\frac{\text{m}}{\text{s}}$ and collides with a 4 kg block initially traveling to the left with an initial speed of 2 $\frac{\text{m}}{\text{s}}$. The 2 kg and 4 kg blocks stick together after the collision.



(a) How fast are the two blocks moving after the collision?

Solution: Let motion to the right be positive, and motion of the left be negative. Using conservation of momentum we get:

$$\begin{aligned}\sum \vec{p_0} &= \sum \vec{p_\text{f}}\\ (2\text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right) + (4\text{ kg})\left(-2\ \frac{\text{m}}{\text{s}}\right) &= (6\text{ kg})v_\text{f}\\ 12\text{ kg}\frac{\text{m}}{\text{s}} &= (6\text{ kg})v_\text{f}\\ v_\text{f} = &= 2\ \frac{\text{m}}{\text{s}}\end{aligned}$$

(b) In what direction are the two blocks moving after the collision?

Solution: The blocks are moving to the right since the final velocity if positive.

(c) What was the impulse exerted on the 4 kg block?

Solution: The Impulse-Momentum Theorem says that the impulse exerted on the 4 kg block equals the change in its momentum.

$$\begin{aligned}J &= \Delta p\\ &= m(v_\text{f} - v_0)\\ &= (4\text{ kg})\left(2\ \frac{\text{m}}{\text{s}} - (-2)\right)\\ &= 16\text{ kg}\frac{\text{m}}{\text{s}}\end{aligned}$$

(d) What was the impulse exerted on the 2 kg block? (You do not need to show any work to answer this problem.)

Solution: According to Newton's Third Law, the impulse exerted on the 2 kg box will be $-16\text{ kg}\frac{\text{m}}{\text{s}}$.

(e) Suppose the two boxes are in contact for only 0.02 s during the collision. What is the magnitude of the average force exerted by the 2 kg block on the 4 kg block?

Solution: We can find the average force using the formula: $J = F_\text{avg}\Delta t$.

$$\begin{aligned}F_\text{avg} &= \frac{J}{\Delta t}\\ &= 1600\text{ N}\end{aligned}$$

(f) What type of collision is this?

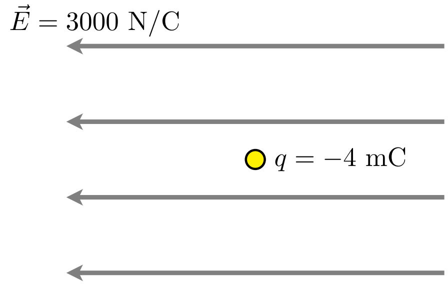
Solution: Because the objects stick together after the collision, it's perfectly inelastic.

(g) How much energy is lost during the collision?

Solution:

$$
\begin{aligned}
\Delta E &= E_\text{f} - E_0 \\
&= \frac{1}{2}(2+4\text{ kg})\left(2\ \frac{\text{m}}{\text{s}}\right)^2 - \left[\frac{1}{2}(2\text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(4\text{ kg})\left(2\ \frac{\text{m}}{\text{s}}\right)^2\right] \\
&= 12\text{ J} - 108\text{ J} \\
&= -96\text{ J}
\end{aligned}
$$

3.
A 3 kg object with a charge of -4 mC is placed in a 3000 N/C electric field as shown in the figure below.



(a) In what direction will the electrostatic force point?

Solution: Because the charge is negative, the force will point in the opposite direction as the field. So, the force points to the right.

(b) What is the magnitude of the electrostatic force?

Solution:

$$\begin{aligned} F &= q\,E \\ &= (4(10^{-3})\ \mathrm{C})\left(3000\ \frac{\mathrm{N}}{\mathrm{C}}\right) \\ &= 12\ \mathrm{N} \end{aligned}$$

(c) If the object starts from rest, how fast will it be moving after it travels 2 m?

Solution: There are many way to calculate the final speed. One way is to use Newton's Second Law to find the acceleration, then plug this into the appropriate kinematics equation to solve for v_{f}. From Newton's Second Law, we see that: $a = F/m = 4\,\mathrm{m/s^2}$. Plugging this into $v_{\mathrm{f}}^2 = v_0^2 + 2\,a\,\Delta x$ gives:

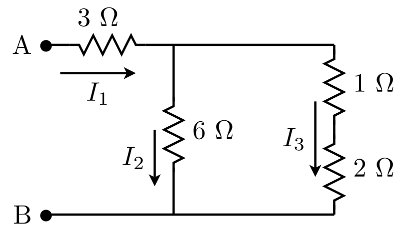
$$\begin{aligned} v_{\mathrm{f}}^2 &= \cancelto{0}{v_0^2} + 2\,a\,\Delta x \\ v_{\mathrm{f}}^2 &= 2\,a\,\Delta x \\ v_{\mathrm{f}} &= \sqrt{2\,a\,\Delta x} \\ &= \sqrt{2\left(4\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(2\ \mathrm{m})} \\ &= 4\ \frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

(d) Through what change in voltage did the object move in part (c)?

Solution: We can relate the change in voltage to the change in energy using $\Delta PE = q\,\Delta V$. Combining this with conservation of energy gives:

$$
\begin{aligned}
\Delta E_{\text{total}} &= 0 \\
\Delta KE + \Delta PE &= 0 \\
\Delta PE &= -\Delta KE \\
q\,\Delta V &= \frac{1}{2} m\,v^2 \\
\Delta V &= \frac{m\,v^2}{2\,q} \\
&= \frac{(3\ \text{kg})\left(4\ \frac{\text{m}}{\text{s}}\right)^2}{2(-4(10^{-3})\ \text{C})} \\
&= -6(10^3)\ \text{V}
\end{aligned}
$$

4. Consider the circuit shown below.



(a) What is the equivalent resistance between points A and B?

Solution: The 1 Ω and 2 Ω resistors on the far right side of the circuit are connected in series. Thus, we can replace these two resistors with a single equivalent resistor of:

$$\begin{aligned} R_{\text{eq}} &= R_1 + R_2 \\ &= (1\ \Omega) + (2\ \Omega) \\ &= 3\ \Omega \end{aligned}$$

The 3 Ω equivalent resistor on the right side of the circuit is connected in parallel with the 6 Ω resistor in the center of the circuit. The equivalent resistance of these two resistors is:

$$\begin{aligned} R_{\text{eq}} &= \frac{R_1 R_2}{R_1 + R_2} \\ &= \frac{(3\ \Omega)(6\ \Omega)}{(3\ \Omega) + (6\ \Omega)} \\ &= 2\ \Omega \end{aligned}$$

Finally, this 2 Ω equivalent resistor is in series with the 3 Ω resistor on the top left. So the overall equivalent resistance is:

$$\begin{aligned} R_{\text{eq}} &= R_1 + R_2 \\ &= (2\ \Omega) + (3\ \Omega) \\ &= 5\ \Omega \end{aligned}$$

(b) Suppose a 10 V battery is connected between points A and B. Calculate I_1, I_2, and I_3 when a 10 V battery is connected.

Solution: Notice that all the current passes through the 3 Ω resistor. Therefore, I_1 is the total current in the circuit. Since we already know the equivalent resistance, we can easily use this to calculate I_1.

$$\begin{aligned} V &= I_{\text{total}}\, R_{\text{eq}} \\ V &= I_1\, R_{\text{eq}} \\ I_1 &= \frac{V}{R_{\text{eq}}} \\ &= \frac{10\ \text{V}}{5\ \Omega} \\ &= 2\ \text{A} \end{aligned}$$

Now that we know I_1, we can calculate I_2 and I_3 using the Loop Rule and the Junction Rule. Using the Junction Rule, we get:

$$\begin{aligned} I_{\text{in}} &= I_{\text{out}} \\ I_1 &= I_2 + I_3 \end{aligned}$$

Applying the Loop Rule to the loop on the right side of the circuit (and working clockwise) gives:

$$\begin{aligned} \Delta V_{\text{loop}} &= 0 \\ 6I_2 - 3I_3 &= 0 \\ I_3 &= 2I_2 \end{aligned}$$

Plugging the equation above into the equation we found from the Junction Rule allows us to solve for I_2:

$$\begin{aligned} I_1 &= I_2 + I_3 \\ I_1 &= I_2 + 2I_2 \\ 2\text{ A} &= 3I_2 \\ I_2 &= 0.67\text{ A} \end{aligned}$$

Finally, plugging this back into $I_3 = 2I_2$ gives $I_3 = 1.33$ A.